

Application of Dimensional Analysis to Predict Airplane Stopping Distance

Mahinder K. Wahi*

Boeing Commercial Airplane Company, Renton, Wash.

A new technique has been developed to predict airplane braking distance performance in terms of aircraft parameters, runway environment, and brake control systems performance. A dimensional analysis technique was used to determine significant dimensionless groups or pi terms required to express the braking performance in equation form. A hardware-analog brake control simulator was used to gather experimental data necessary to determine the constants and exponents in the performance equation. Among the airplanes studied were the Boeing 727, 737, 747, the Lockheed C-141, and the McDonnell F-4. A prediction equation permits the calculation of stopping distance, assuming that proper information concerning airplane and weather parameters is available, as well as accurately measured tire-runway friction coefficients.

Nomenclature

C_D	= coefficient of drag
C_L	= coefficient of lift
F_e	= engine idle thrust
F_{e0}	= engine idle thrust at zero velocity
g	= acceleration caused by gravity
KE	= change in idle thrust with velocity
s	= braking distance
V_w	= headwind or tailwind velocity
W	= airplane landing weight
μ, μ_s	= peak available friction coefficient
η_s	= braking distance efficiency
π	= pi term
ρ	= air density

Introduction

MEANINGFUL braking distances can be determined only with an airplane test or a brake control simulation. Because airplane testing is expensive and time-consuming, the simulator can be used as an effective tool to analyze sensitivity of each parameter under controlled conditions. The simulator also allows studying the braking performance under extreme conditions that cannot be duplicated in actual aircraft tests for safety reasons. Results of a series of carefully planned simulator tests were utilized in a dimensional analysis to develop a braking distance prediction equation.

Development of Prediction Model

Dimensional analysis differs from other types of analysis in that it is based solely on the relationships that must exist among the pertinent variables because of their dimensions rather than the laws of physics and classical mathematical derivation. In itself, dimensional analysis gives qualitative rather than quantitative relationships, but, when combined with experimental procedures, it may be made to supply quantitative results and accurate prediction equations.

Two general methods are available for developing prediction equations. One method consists of establishing, by careful observation and measurement, the effect of the pertinent variables upon the quantity to be predicted. The other method consists of applying the natural laws pertinent to the problem to develop relationships among the significant variables. The natural laws used in this method are simply

generalizations of reliable information assembled through observation and measurement. The first method usually is referred to as the experimental method, the second as the analytical method. However, each involves analysis, and each is basically dependent upon experimental findings. Often, it is necessary to develop prediction equations for phenomena to which the usual analytical procedures are not well adapted. Braking distance performance is one such problem.

Figure 1 is a flow chart where each block represents a major step of analysis in the formulation of the prediction equation. After the variables affecting aircraft stopping performance had been identified, an analog-hardware brake control simulator^{1,2} was utilized to conduct a parametric study of these variables on the brake control systems of five airplanes, namely, the Boeing 727, 737, 747, the Lockheed C-141, and the McDonnell F-4.

A grading system was established to enable categorizing variables in order of their significance. It was decided to consider all parameters effecting a 2% change in the baseline braking distance caused by realistic change in any independent parameter within the operational envelope of that airplane. (The repeatability of the simulator itself results in 1% variations.) This list, shown in Table 1, was evaluated and trimmed down to pertinent variables (explained later) to be considered for dimensional analysis. Using conventional dimensional analysis techniques,^{3,4} the number and form of the dimensionless groups, or pi terms, were obtained. Statistical curve-fitting programs were used⁵ to obtain the component equations (relationship between any two π terms while others are held constant), which later were combined to yield the final prediction equation. The prediction equations were validated by meeting the necessary and sufficient conditions generated during analysis. A correlation (with simulator data) study then was completed, and a prediction accuracy of $\pm 5\%$ was achieved. This process was repeated for each of the five airplanes.

Determination of π Terms

The first step in forming a prediction model is to identify the pertinent and independent variables. This step is by far the most important because the validity of the results depends on the correctness with which the pertinent factors are selected. This required combining of some of the interdependent variables listed in Table 1, e.g., 3, 4, and 6. In addition, parameters that are pilot-technique-dependent, e.g., 5, 7, 8, and 11 (and outside the scope of the present work), had to be excluded. The list of resultant independent variables is shown in Table 2.

The second step is to express the dependent variable, in this case stopping distance s , as a function of the independent

Received Dec. 18, 1975; revision received June 16, 1976.

Index categories: Aircraft Deceleration Systems; Aircraft Landing Dynamics; Computer Technology and Computer Simulation Techniques.

*Senior Engineer, Landing Gear Research.

As a starting point, a baseline reference was defined for each aircraft. The baseline airplane represents an aircraft in three-point taxi attitude and of typical (midrange) landing weight, approach speed, center-of-gravity location, landing flap setting, and engine thrust. The actual parameters required for the airplane simulator are defined in Ref. 2. Table 3 lists the baseline values used for two of the five airplanes.

Table 3 Baseline values used in airplane simulation and prediction model

Airplane parameter Symbol	Units	Airplane	
		747-200	F-4E
C_D	...	0.18	0.32
C_L	...	0.67	0.27
F_{e0}	lbf	9480	1260
KE	lbf-sec/ft	17.1	4.98
ρ	lbf-sec ² /ft	0.00238	0.00238
v	fps	219	256
v_{stop}	fps	24	24
W	lbf	510,000	35,000

planes, 747 (a transport) and F-4 (a fighter). For reasons of brevity, data for the other three transport airplanes have not been included. Detailed data and analysis can be found in Ref. 2.

The data used in the brake control simulator for the Boeing airplanes (727, 737, 747) are well documented within the Boeing Company. They represent a compilation of the currently accepted data used in various Boeing airplane simulators and were checked for accuracy to insure the reliability of the results obtained from the computer study. Data for the F-4 and the C-141 were obtained from the U. S. Air Force.

During the sensitivity study, each parameter was changed, and the new value of braking distance was evaluated on the simulator. The range over which a parameter was varied reflected values observed in normal service of the airplane. Some of the variables were not independent, and those groups of interrelated parameters were varied appropriately together. An example of this is stall speed and gross weight. Tables 4 and 5 show the data obtained from the parametric study just described.

The best procedure for evaluating a function is to arrange the observations so that all but one of the pi terms containing the independent variables in the function remain constant. Then the remaining independently variable pi term is varied to establish a relationship between it and the dependent variable (π_1 term). This procedure is repeated for each of the pi terms in the function; the resulting relationships between π_1 and the other individual pi terms are called component equations. Statistical curve-fitting computer programs were used to generate the component equations.

Plots were prepared of π_1 vs π_2 , π_1 vs π_3 , and π_1 vs π_4 for each airplane. This helped determine the general form of relationship that could exist between π_1 and π_2 and so on; e.g., the π_1 vs π_2 and π_1 vs π_4 data plotted as straight lines on the log-log paper and thus should have a relationship of the form $y = Ax^B$, where A is a constant and B a polynomial. However, π_1 vs π_3 data plotted as curves even on log-log

Table 4 Parametric study data

Test condition and airplane parameter changed	Airplane	
	747-200	F-4E
1a) Maximum landing weight		
W , lb	564,000	46,000
v , fps	231	292
1b) Minimum landing weight		
W , lb	400,000	30,000
v , fps	194	237
3a) Brake application speed + 10%		
v , fps	230	282
3b) Brake application speed + 20%		
v , fps	241	308
4c) No spoiler or drag device		
C_L	1.15	0.27
C_D	0.1317	0.11
4d) 60% effective spoilers		
C_L	0.862	0.27
C_D	0.1607	0.228
4e) 40% effective spoilers		
C_L	0.958	0.27
C_D	0.151	0.192
4f) 120% engine idle thrust		
F_{e0} , lb	11,375	1510
KE , ft-lb	20.5	-4.0
4g) 80% engine idle thrust		
F_{e0} , lb	7854	1010
KE , ft-lb	13.7	-6.0

Table 5 Simulator braking distance results [stopping distance (braking segment only), ft]

Condition	747			F-4		
Available μ	0.6	0.4	0.2	0.6	0.4	0.2
Baseline	1,905	2,630	4,598	2,766	3,974	8,593
Max. wt.	2,090	2,891	5,040	3,255	4,880	8,832
Min. wt.	1,528	2,137	3,774	2,704	3,841	8,271
+ 10% v	2,112 ^a	2,910 ^a	5,036 ^a	3,370	4,566	8,825
+ 20% v	2,334 ^b	3,201 ^b	5,469 ^b	3,907	5,022	8,885
60% SP	2,085	2,908	5,135	3,215	4,850	10,738
40% SP	2,199	3,066	5,464	3,427	5,326	12,274
No SP	2,507	3,491	6,262	4,095	7,061	...
120% F_e	1,916	2,661	4,702	2,893	4,196	10,615
80% F_e	1,868	2,580	4,438	2,722	3,836	7,218

^a + 5%, ^b + 10%.

Table 6 Summary of component equations

Airplane model	μ^a	Equation	Equation number
747	0.6	$(\pi_1) = 0.838(\pi_2)^{-0.815}$	(6a)
		$(\pi_1) = 1.2214(\pi_3)^{[0.14803 - 0.11286\%SP^b]}$	(6b)
		$(\pi_1) = 1.5162(\pi_4)^{-0.01676}$	(6c)
	0.4	$(\pi_1) = 0.838(\pi_2)^{-0.815}$	(6a)
		$(\pi_1) = 1.5541(\pi_3)^{[0.18957 - 0.09193\%SP]}$	(6d)
		$(\pi_1) = 2.3966(\pi_4)^{-0.03004}$	(6e)
	0.2	$(\pi_1) = 0.838(\pi_2)^{-0.815}$	(6a)
		$(\pi_1) = 2.5185(\pi_3)^{[0.2365 - 0.0818\%SP]}$	(6f)
		$(\pi_1) = 5.4099(\pi_4)^{-0.05588}$	(6g)
F-4	0.6	$(\pi_1) = 0.84575(\pi_2)^{-0.9239}$	(6h)
		$(\pi_1) = 1.4699(\pi_3)^{+0.3636}$	(6i)
		$(\pi_1) = 2.5323(\pi_4)^{-0.04408}$	(6j)
	0.4	$(\pi_1) = 0.84575(\pi_2)^{-0.9239}$	(6h)
		$(\pi_1) = 2.1615(\pi_3)^{[0.52698 + 0.08287\%SP]}$	(6k)
		$(\pi_1) = 5.8972(\pi_4)^{-0.07895}$	(6l)
	0.2	$(\pi_1) = 0.7337(\pi_2)^{-1.0694}$	(6m)
		$(\pi_1) = 4.7285(\pi_3)^{+0.69738}$	(6n)
		$(\pi_1) = 161.9648(\pi_4)^{-0.26017}$	(6o)

^a Value of μ used in the data set.^b %SP is the percentage of the spoiler configuration.

Table 7 Test of validity for the function to be a product

Airplane model	Value of function in Eq. (6)			Ideal value of function in Eq. (6)	Deviation percentage		
	LHS ^a	RHS	RHS		LHS	RHS	RHS
	$\bar{\pi}_2 = 0.6$	$\bar{\pi}_2 = 0.4$	$\bar{\pi}_2 = 0.2$		$\bar{\pi}_2 = 0.6$	$\bar{\pi}_2 = 0.4$	$\bar{\pi}_2 = 0.2$
747	1.008	1.002	0.996	1.0	+0.8	+0.2	-0.4
F-4	1.010	0.994	0.985	1.0	+1.0	-0.5	-1.5

^aLHS = left-hand side; RHS = right-hand side.

Table 8 Test of validity for constant term

Airplane model	$C = [1/F(\bar{\pi}_2, \bar{\pi}_3, \bar{\pi}_4)]^2$			Ideal value of C	Average deviation, %	Value of $\bar{\pi}_2$
	Component equation used					
	π_1 vs π_2	π_1 vs π_3	π_1 vs π_4			
747	0.6190	0.6113	0.6075	0.6113	+0.2	0.6
	0.3199	0.3206	0.3174	0.3206	-0.4	0.4
	0.1033	0.1049	0.1043	0.1049	-0.7	0.2
F-4	0.5430	0.5228	0.5336	0.5414	-1.5	0.6
	0.2571	0.2624	0.2603	0.2624	-0.9	0.4
	...	0.05647	0.05426	0.0561	-1.2	0.2

paper, and it was necessary to use multiple linear regression to arrive at the desired component equation form. A summary of component equations is listed in Table 6.

Prediction Equations

When the computer equations have been determined, they are combined in a certain manner to give a general relationship. It is possible for some of the component equations to be combined by multiplication, whereas others require addition in the formation of the resultant prediction equation. In general, these two methods are adequate for the majority of engineering problems. For the stopping distance problem, the analysis showed that the prediction equation should be formed by multiplication. The necessary and sufficient conditions to be met for the function to be a product were developed and translated into tests of validity.

It can be shown that, when four pi terms are involved and the component equations are combined by multiplication, the prediction equation is of the form³

$$(\pi_1) = (C) (\pi_1)_{\bar{\pi}_2, \bar{\pi}_3, \bar{\pi}_4} (\pi_1)_{\bar{\pi}_2, \bar{\pi}_3} (\pi_1)_{\bar{\pi}_2, \bar{\pi}_4} \quad (3)$$

where $(\pi_1)_{\bar{\pi}_2, \bar{\pi}_3, \bar{\pi}_4}$ means variation of π_2 while π_3 and π_4 are held constant.

The analysis shows that the value of the constant term C is of the form

$$C = 1/[F(\bar{\pi}_2, \bar{\pi}_3, \bar{\pi}_4)]^2 \quad (4)$$

Thus the prediction equation is of the form

$$F(\pi_2, \pi_3, \pi_4) = \frac{F(\pi_2, \bar{\pi}_3, \bar{\pi}_4)F(\bar{\pi}_2, \pi_3, \bar{\pi}_4)F(\bar{\pi}_2, \bar{\pi}_3, \pi_4)}{[F(\bar{\pi}_2, \bar{\pi}_3, \bar{\pi}_4)]^2} \quad (5)$$

The equations constituting a test for the validity of Eq. (5) can be shown to be³

$$\frac{F(\bar{\pi}_2, \pi_3, \bar{\pi}_4)F(\bar{\pi}_2, \bar{\pi}_3, \pi_4)}{[F(\bar{\pi}_2, \bar{\pi}_3, \bar{\pi}_4)]^2} = \frac{F(\bar{\pi}_2, \pi_3, \bar{\pi}_4)F(\bar{\pi}_2, \bar{\pi}_3, \pi_4)}{[F(\bar{\pi}_2, \bar{\pi}_3, \bar{\pi}_4)]^2} \quad (6)$$

or

$$\frac{F(\pi_2, \bar{\pi}_3, \bar{\pi}_4)F(\bar{\pi}_2, \bar{\pi}_3, \pi_4)}{[F(\bar{\pi}_2, \bar{\pi}_3, \bar{\pi}_4)]^2} = \frac{F(\pi_2, \bar{\pi}_3, \bar{\pi}_4)F(\bar{\pi}_2, \bar{\pi}_3, \pi_4)}{[F(\bar{\pi}_2, \bar{\pi}_3, \bar{\pi}_4)]^2}$$

Table 9 Summary of prediction equations

Airplane model	μ^*	Equation	Equation number
747	0.6	$(\pi_1) = 0.9539(\pi_2) - 0.815(\pi_3) [0.14803 - 0.11286\%SP] (\pi_4) - 0.01676$	(9a)
	0.4	$(\pi_1) = 0.9974(\pi_2) - 0.815(\pi_3) [0.18957 - 0.09193\%SP] (\pi_4) - 0.03004$	(9b)
	0.2	$(\pi_1) = 1.1897(\pi_2) - 0.815(\pi_3) [0.2365 - 0.0818\%SP] (\pi_4) - 0.05588$	(9c)
F-4	0.6	$(\pi_1) = 1.6790(\pi_2) - 0.9239(\pi_3) 0.3636(\pi_4) - 0.04408$	(9d)
	0.4	$(\pi_1) = 2.80256(\pi_2) - 0.9239(\pi_3) [0.52698 + 0.08287\%SP] (\pi_4) - 0.07895$	(9e)
	0.2	$(\pi_1) = 31.52(\pi_2) - 1.0694(\pi_3) 0.69738(\pi_4) - 0.26017$	(9f)

Table 10 Summary of percentage errors

Airplane model	Using equation	Applied to data at $\bar{\pi}_2$	Error range, %
747	(9a)	0.6	-1.7 to +20
		0.4	-1.3 to +2.5
		0.2	-1.8 to +4.8
	(9b)	0.6	-3.2 to +0.9
		0.4	-1.8 to +1.6
		0.2	-1.6 to +3.9
F-4	(9c)	0.6	-5.0 to +2.4
		0.4	-3.9 to +2.3
		0.2	-1.1 to +2.6
	(9d)	0.6	-2.9 to +0.7
		0.4	-5.6 to +5.0
		0.2	-6.7 to +7.3
	(9e)	0.4	-5.4 to +5.4
		0.2	-1.7 to +3.3

The values $\bar{\pi}_2$ and $\bar{\pi}_3$ are values of π_2 and π_3 held constant at some value other than $\bar{\pi}_2$ and $\bar{\pi}_3$. Thus, from the observed data, $\bar{\pi}_2 = 0.6$ (the primary set of data, for example), and $\bar{\pi}_2 = 0.4$, $\bar{\pi}_2 = 0.2$ (supplementary sets of data).

If the supplementary sets of data satisfy either Eq. (6) or (6a), the general equation can be formed by multiplying the component equations together and dividing by the constant, as indicated in Eq. (5). This test was applied to all available data (component equations); the results are shown in Table 7, clearly indicating the validity of the approach.

Another test of validity was to calculate the value of the constant term C of Eq. (3). The test requires that any of the three component equations should yield an identical value for C . This test also was applied to all of the test data; the results are shown in Table 8. Again, the accuracy achieved is satisfactory.

The two validity tests were successful, thus permitting the writing of the prediction equations. A summary of all prediction equations is listed in Table 9. Equation (9a) is a combination of Eqs. (6a-6c) and corresponding C . Equation (9b) is a combination of Eqs. (6a, 6d, 6e), and corresponding C , and so on.

Model to Simulator Correlation

The prediction equations were used next to correlate back with the stopping distance data collected on the simulator. A summary of errors in correlation is listed in Table 10. The limitations (range of validity) of the prediction equations are as follows:

1) Equations (9a-9c) are applicable for μ values of 0.1 to 0.6.

2) Equations (9d) and (9e) are applicable for μ values of 0.3 to 0.6.

3) Equation (9f) is applicable for μ values of 0.1 to 0.2 only.

For a given model, the three prediction equations, e.g., Eqs. (9a-9c), are interchangeable, alternate solutions, since their range of applicability and validity is common. However, Eq. (9f) is a unique solution and not interchangeable with its counterpart, Eq. (9d) or (9e). Some airplane systems need only one prediction equation to define the entire range of μ values tested on the simulator; others need more than one

Fig. 3 Mu-efficiency curves.

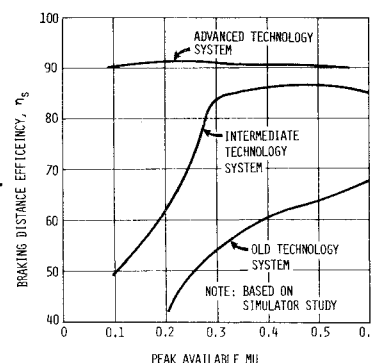
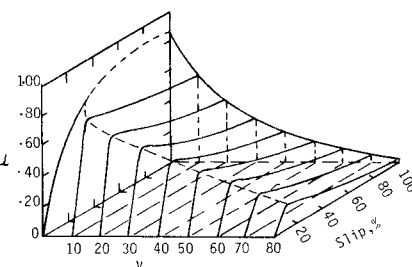


Fig. 4 Mu-velocity curve for wet runway.



equation. The reason for this can be comprehended by studying braking distance efficiency curves for the various systems, as shown in Fig. 3.

A study of Fig. 3 shows that the curve for the advanced technology system is nearly linear; sharp changes in slope appear in the curves for the old as well as the intermediate technology systems at μ values of 0.3. Piecemeal linearization always is required when writing mathematical relationships for curves of the type shown for the old and intermediate technology systems. That is why more than one prediction equation was necessary to treat these two cases. The correlation data error summary (Table 10) indicates that, for almost all conditions, a prediction accuracy of $\pm 5\%$ can be achieved. The error ranges for the other three airplanes were all within $\pm 5\%$.

Wet Runway Analysis

During experimental tests, a wet runway was simulated so that the available ground μ was programmed to vary with speed (see Fig. 4). The μ values (end points) used were 0.05 at brake application speed and 0.5 at the end of the stop. The average value of peak available μ for the braking system was unknown, and so it was decided to use the component equations formed earlier (the π_1 vs π_2 relationships) to calculate peak available μ . Based on calculations for wet runways, prediction equations were generated for wet runway cases. Again, a correlation prediction accuracy of $\pm 5\%$ was obtained. This gave additional confidence to the selected methodology for forming prediction equations.

Conclusions

The formulation of the prediction equation has been accomplished with the use of dimensional analysis. The effect of

parameter variations on stopping distance was examined on the brake control simulator and compared to baseline data. Parameters that changed the stopping distance by more than 2% were considered significant and included in the dimensional analysis. The resulting prediction equation appears with the general format

$$(sg/v^2) = C(\mu)^\alpha (C_L/C_D)^\beta (\rho v^6 / Feg^2)^\delta$$

The exponents α , β , δ and the coefficient C result from the sensitivity study and the dimensional analysis. For each airplane, a unique coefficient and a set of exponents exist.

The equation permits the calculation of the airplane braking distance, assuming that proper information of airplane and weather parameters and an accurate and meaningful measurement or prediction of the tire-runway friction coefficient are available. The most elusive value in the equation is that of the tire-runway friction coefficient. Efforts now are underway⁹ to formulate a tire model that could be used to correlate measured interface friction from one tire to another under various conditions of operation. This is the peak value developed under the dynamic conditions of braking and is represented in the hardware-analog simulation with a friction slip curve. Hence the following conclusions are reached:

1) A dimensional analysis technique successfully can express braking phenomena in the form of a mathematical (model) equation.

2) Experimental data from an airplane braking distance sensitivity study are needed to determine the constants and exponents in the model equation leading to a prediction model equation.

3) With the proper information of airplane and weather parameters and an accurate and meaningful measurement of

the tire-runway friction coefficient, airplane braking distances can be predicted within reasonable tolerances.

Acknowledgment

This work was conducted under the combined NASA, USAF, and FAA Contract F33657-74-C-0129 entitled Combat Traction II, Phase II, Sensitivity Study. The author acknowledges the very substantial assistance and contribution of S. Warren, H. Straub, N. Attri, and R. Amberg of the Boeing Company in completion of this project.

References

- ¹Amberg, R. L., "Baseline Simulation for Evaluation of Brake Antiskid Systems," The Boeing Co., Seattle, Wash., Doc. D6-58384-3TN, 1969, revised 1974.
- ²Wahi, M. K., et al., "Combat Traction II, Phase II, Sensitivity Study," Aeronautical Systems Div., Wright Patterson AFB, Ohio, ASD-TR-74-41, Vols. I, II, Oct. 1974.
- ³Murphy, G. C., *Similitude in Engineering*, Ronald Press Co., New York, 1950.
- ⁴Baker, W. E. and Wilfred, J. S., *Similarity Methods in Engineering Dynamic Theory and Practice of Scale Modeling*, Spartan Books, 1973.
- ⁵"CTS Statistical Package (STAT-PK)," Boeing Computer Services (BCS), 1974.
- ⁶Yager, T. J., et al., "A Comparison of Aircraft and Ground Vehicle Stopping Performance on Dry, Wet, Flooded, Slush-Snow, and Ice-Covered Runways," NASA TN D-6098, Nov. 1970.
- ⁷Horne, W. B., et al., "Traction Measurements of Several Runways Under Wet and Dry Conditions with a Boeing 727, a Diagonal-Braked Vehicle, and a MU-Meter," LWP-1016, Dec. 1971.
- ⁸Tracy, W. V., "F-4 Rain Tire Project Technical Report," Aeronautical Systems Div., Wright Patterson AFB, Ohio, ASD-TR-74-37, Oct. 1974.
- ⁹"Combat Traction II, Phase II," Contract F33657-74-C-0129 (extended), May 1975, U. S. Air Force.